Evaluation of Relative Clock Stability in Cellular Networks

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BIOGRAPHIES
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ABSTRACT
The relative stability of base transceiver station (BTS) clocks in cellular networks is studied. It is demonstrated that although BTS clocks are not perfectly synchronized to GPS, they are relatively stable with respect to each other, permitting exclusive navigation with cellular signals in the absence of global navigation satellite system (GNSS) signals. A navigation observable is developed leveraging the relative stability of BTS clocks and a stochastic dynamical model for the clock deviations is proposed. Experimental data to multiple BTSs over twenty-four hours are presented.

I. INTRODUCTION
Global navigation satellite system (GNSS) has been at the heart of most vehicular and personal navigation systems. The limitations of GNSS are well-known: the signals are unusable in indoor environments, unreliable in deep urban canyons, are susceptible to unintentional and intentional jamming, and are spoofable [1]. These limitations necessitate backup and alternative navigation systems to GNSS. Such systems can be sensor-based (e.g., inertial navigation system (INS) [2], vision [3], or lidar [4]) or signal-based (e.g., dedicated beacons [5] or signals of opportunity (SOPs) [6]).

SOPs are ambient radio signals in the environment which are not intended for navigation, such as AM/FM [7, 8], cellular [9] [10], Wi-Fi [11,12], television [13,14], and iridium satellites [15,16]. A number of experimental results have demonstrated receiver localization and timing via SOPs [14,17–19] while other studies answered theoretical questions on the observability and estimability of the SOPs landscape for a different number of receivers, a different number of SOPs, and various a priori knowledge scenarios [20]. Cellular signals are particularly attractive SOPs for navigation for three main reasons. First, they are abundant and are available at a favorable horizontal geometry by virtue of the cellular system design. Second, the received signal power in cellular signals is much higher than GNSS, making them easier to acquire and track. Third, cellular signals are free to exploit by listening to the pilot and synchronization signals broadcast by the base transceiver stations (BTSs), uncompromising the privacy of the navigating receiver. However, there are three main challenges associated with using cellular signals for navigation. First, unlike GNSS space vehicles that transmit their clock and orbital parameters from which their position and clock error states can be deduced, BTSs may not transmit any information about their position nor clock error states. Second, since cellular SOPs are not intended for navigation purposes, specialized receivers must be designed to extract navigation...
observables from such signals. Third, rigorous navigation error budgets and performance analyses under different error sources are not thoroughly studied.

Recent research in navigation using SOPs addressed in part all three challenges for cellular code-division multiple access (CDMA) and long-term evolution (LTE) signals [9, 21]. Moreover, errors inherent to cellular CDMA systems that are not harmful for communication purposes but severely degrade the navigation performance were reported [9, 22]. This paper extends previous work on sources of error characterization for navigation with cellular signals by (1) characterizing the relative stability of BTS clocks for navigation purposes and (2) proposing a framework for navigating with cellular signals using carrier phase measurements. As mentioned earlier, the unknown nature of the BTS states poses a challenge to the navigating receiver. While the position of the BTS is constant, its clock bias is stochastic and dynamic, and hence must be continuously estimated. However, a closer look at the clock error states of neighboring BTSs computed from carrier phase measurements shows that although they are drifting from GPS time, their clock biases appear to be synchronized up to a constant offset. Consequently, if it is not required that the receiver estimates its clock bias with respect to an absolute time reference (e.g., GPS time), then it can solve for its position state knowing only the position of the BTSs as well as its own initial position. This navigation framework; however, invokes some approximations justified by the observed relative stability of those BTSs’ clocks.

The rest of the paper is organized as follows. Section II describes the carrier phase measurement model and the pseudorange model parameterized by the receiver and BTS states. Section III investigates the relative clock stability of neighboring cellular BTSs and proposes a navigation framework using a modified pseudorange model. Concluding remarks are given in Section IV.

II. MODEL DESCRIPTION

In this section, the carrier phase observable, the receiver and BTS states, and the pseudorange model are presented.

A. Carrier Phase Observable

In cellular CDMA systems, a pilot signal consisting of a pseudorandom noise sequence, called the short code, modulated by a carrier signal is broadcast by each BTS. Therefore, by knowing the shortcode, the receiver may measure the code phase of the pilot signal as well as its carrier phase, hence forming a pseudorange measurement to each BTS transmitting the pilot signal. In the rest of the paper, the cellular CDMA module of the Multichannel Adaptive TRansceiver Information eXtractor (MATRIX) software-defined radio (SDR) developed at the Autonomous Systems Perception, Intelligence, and Navigation (ASPIN) Laboratory at the University of California, Riverside (UCR) is employed to extract code phase and Doppler frequency information from the received cellular CDMA pilot signals [9, 23]. By integrating the Doppler measurement over time, the continuous-time carrier phase measurement (expressed in cycles) [24] from the $i$th BTS, denoted $\phi_i(t)$, may be obtained according to

$$\phi_i(t) = \phi_i(t_0) + \int_{t_0}^{t} f_{D_i}(\tau)d\tau, \quad i = 1, \ldots, N,$$

where $f_{D_i}$ is the Doppler measurement for the $i$th BTS, $\phi_i(t_0)$ is the initial carrier phase, and $N$ is the number of BTSs in the environment. Assuming a constant Doppler during a subaccumulation period $T$, the expression above can be discretized to yield

$$\phi_i(t_k) = \phi_i(t_0) + \sum_{l=0}^{k-1} f_{D_i}(t_l)T,$$

where $t_k \triangleq t_0 + kT$. In what follows, the time argument $t_k$ will be replaced by $k$ for simplicity of notation. Subsequently, the pseudorange may be obtained by multiplying the carrier phase by the signal’s wavelength $\lambda$, yielding

$$\rho_i(k) = \lambda\phi_i(0) + \lambda T \sum_{l=0}^{k-1} f_{D_i}(l) = \rho_i(0) + \Delta\rho_i(k),$$

(1)
where \( \rho_i(0) \triangleq \lambda \varphi_i(0) \) is the initial pseudorange and \( \Delta \rho_i(k) \triangleq \lambda T \sum_{l=0}^{k-1} f_{D_i(l)} \) is the delta pseudorange. The initial pseudorange is obtained by multiplying the initial code phase estimate by the speed-of-light.

### B. Receiver and SOP States

The general scenario considered in this paper is as follows. A receiver enters a cellular CDMA environment consisting of \( N \) BTSs. The receiver state \( x_r \) consists of its horizontal position vector \( r_r \triangleq [x_r, y_r]^T \) and its clock error state \( c \delta t_r \) where \( \delta t_r \) is the receiver’s clock bias with respect to GPS time and \( c \) is the speed-of-light, hence

\[
x_r \triangleq [r_r^T, c \delta t_r]^T.
\]

Similarly, the state of the \( i \)th BTS is expressed as

\[
x_{s_i} \triangleq [r_{s_i}^T, c \delta t_{s_i}]^T, \quad i = 1, \ldots, N.
\]

The receiver is drawing pseudorange measurements to the BTSs in view and is assumed to initially have access to GNSS signals. Then, at a certain point in time, GNSS becomes unavailable for some reason (e.g., signal attenuation or jamming). The time index at which GNSS is cut off is set to \( k \equiv 1 \), and the receiver’s position at \( k = 0 \) (the last time index GNSS was available) is known to the receiver. Moreover, pseudorange measurements at \( k = 0 \) to each BTS are available. In this framework, the BTS positions are assumed to be known. Since these BTSs are spatially stationary, they can be mapped \textit{a priori} using multiple receivers (e.g., as discussed in [25, 26]. Subsequently, in the rest of the paper, it is assumed that the receiver has knowledge of the following quantities

\[
\{r_{s_i}\}_{i=1}^{N}, \quad \{\rho_i(k)\}_{i=1}^{N} \quad \forall \, k = 0, 1, 2, \ldots \quad \text{and} \quad r_r(0).
\]

### C. Pseudorange Model

After discretization and some mild approximations [20], the pseudorange in (1) may be expressed as

\[
\rho_i(k) = \|r_r(k) - r_{s_i}\|_2 + c[\delta t_r(k) - \delta s_i(k)] + v_i(k),
\]

where \( v_i \) is the measurement noise, which is modeled a zero-mean white Gaussian variable with variance \( \sigma_i^2 \).

### III. RELATIVE CLOCK STABILITY OF CELLULAR BASE STATIONS

This section investigates the relative stability of BTS clocks using carrier phase measurements. Then, the pseudorange measurement given in (2) is re-parameterized to leverage the relative stability of BTS clocks.

#### A. Motivating Experimental Results

This subsection motivates the forthcoming study experimentally. The setup consists of a receiver mounted on the roof of the Winston Chung Hall at UCR, which was equipped with a high-gain tri-band cellular antenna and a surveyor-grade GPS antenna. The GPS and cellular signals were simultaneously down-mixed and synchronously sampled via a dual-channel universal software radio peripheral (USRP), driven by a GPS-disciplined oscillator (GPSDO). The cellular receiver was tuned to a 883.98 MHz carrier frequency, which is a cellular CDMA channel allocated for the U.S. cellular provider Verizon Wireless. Samples of the received signals were stored for off-line post-processing. The GPS signal was processed by the Generalized Radionavigation Interfusion Device (GRID) SDR [27] and the cellular CDMA signals were processed by the cellular CDMA module of the MATRIX SDR. The receiver sampled GPS and cellular CDMA signals for a period of 24 hours. Fig. 1 shows the experimental setup (the third BTS is not shown).

After processing the GPS and cellular signals, code phase and Doppler frequency measurements to three BTSs in the vicinity of the receiver and the receiver’s clock bias with respect to GPS were obtained. Fig. 2 (a) shows the
delta pseudoranges to the three BTSs as well as the receiver’s clock bias from which its initial value was removed, denoted by

\[
\delta t_r(k) \triangleq \delta t_r(k) - \delta t_r(0).
\]  

Since the receiver is stationary, the drift in the delta pseudoranges can only be attributed to the drift in the BTSs’ clocks. However, it can be seen from Fig. 2 (a) that the delta pseudoranges are dominated by a common term, and the relatively “fast” variations are caused by the receiver’s clock bias. Fig. 2 (b) shows the delta pseudoranges to the three BTSs corrected by \(c\delta t_r(k)\).

Figs. 2 (a)–(b) show that although the BTS clocks are not synchronized to GPS, they are relatively synchronized to one another. This key observation is critical to enabling the receiver to localize itself in the cellular environment.

**B. Pseudorange Re-Parametrization**

Motivated by the experimental results in Fig. 2, the following re-parametrization is proposed

\[
c\delta t_s_i(k) \triangleq c\delta t_s_i(k) - c\delta t_s_i(0) \equiv c\delta t_s(k) - \epsilon_i(k),
\]

where \(c\delta t_s_i(k)\) is a time-varying common bias term and \(\epsilon_i(k)\) is the deviation of \(c\delta t_s_i(k)\) from this common bias. It is convenient to define \(\delta t_s(k)\) and \(\epsilon_i(k)\) as

\[
\delta t_s(k) \triangleq \frac{1}{N} \sum_{i=1}^{N} \bar{\delta} t_s_i(k), \quad \epsilon_i(k) \triangleq c\delta t_s(k) - c\bar{\delta} t_s_i(k).
\]  

Combining (2)–(5) yields

\[
\rho_i(k) = ||r_r(k) - r_s_i||_2 + c[\bar{\delta} t_r(k) - \delta t_s(k)] + c\delta t_0_i + \epsilon_i(k) + v_i(k),
\]
where $\delta t_0 \triangleq \delta t_r(0) - \delta t_s(0)$. Define

$$\delta t(k) \triangleq \delta t_r(k) - \delta t_s(k),$$

then the pseudorange may be expressed as

$$\rho_i(k) = \| r_r(k) - r_s(k) \|_2 + c\delta t(k) + c\delta t_0 + \epsilon_i(k) + v_i(k).$$

Note that $c\delta t_r(0)$ in $c\delta t_0$ is common to all BTSs; hence, it will have no effect on the position estimate. However, $c\delta t_s(0)$ is different for each BTS and may not necessarily be zero. The cellular CDMA standard requires $|\delta t_s(k)|$ to be less than $10$ μs; hence, $\delta t_s(0)$ could be anywhere between -10 and 10 μs [28]. In practice, this initial bias has been observed to be between -3 and 3 μs for U.S. carriers, which is still unacceptable for navigation purposes. Therefore, it is crucial to know $c\delta t_0$. Since $r_r(0)$ is known, the initial bias $c\delta t_0$ may be estimated according to

$$c\delta t_0 \approx \rho_i(0) - \| r_r(0) - r_s(k) \|_2.$$  

This approximation ignores the contribution of the initial measurement noise. If the receiver is initially stationary for a period $LT$ seconds, which is short enough such that $\delta t(k) \approx 0$ for $k = 1, \ldots, L$, then the $L$ first samples may be averaged to obtain a more accurate estimate of $c\delta t_0$. The term $\epsilon_i(k)$ is assumed to be part of the measurement noise. Subsequently, the receiver only needs to estimate its position $r_r(k)$ and the common clock bias $c\delta t(k)$. Next, $\epsilon_i(k)$ is modeled as a stochastic sequence.

### C. Stochastic Modeling of Clock Deviations

As discussed in the previous subsection, $\epsilon_i(k)$ is regarded as an additional source of error. Therefore, it is important to know the magnitude of these deviations and how they evolve with time. Fig. 3 shows the clock bias deviations $\epsilon_i(k)$ for the three BTSs over the 24-hour period. While the whiteness of $\{\epsilon_i(k)\}_{i=1}^3$ cannot be straightforwardly established, it can be seen that they are bounded with $|\epsilon_i(k)| < 4m \forall i$.

![Fig. 3. Plot of the deviations $\epsilon_i(k)$ from the common clock bias for three BTSs over 24 hours.](image)

It is hypothesized that $\epsilon_i$ adheres to an $n$th order auto-regressive model, according to

$$\epsilon_i(k + 1) - \sum_{l=1}^{n} \phi_l \epsilon_i(k + 1 - l) = w(k),$$

where $\{\phi_l\}_{l=1}^n$ are the auto-regression coefficients and $w(k)$ is a random white sequence. Using system identification tools, one can find the order $n$ and corresponding $\{\phi_l\}_{l=1}^n$. Finding a minimal realization of this sequence is a future research question. However, it was noticed from experimental data that these sequences are stable, and hence the variance of these sequences will reach a steady-state value denoted by $\sigma_i^2$. Subsequently, the modified measurement is modeled as follows

$$\rho_i(k) = \| r_r(k) - r_s(k) \|_2 + c\delta t(k) + c\delta t_0 + \eta_i(k).$$  \hspace{1cm} (6)

where $\eta_i(k) \triangleq \epsilon_i(k) + v_i(k)$ is modeled as a white Gaussian sequence with variance $\sigma_i^2 + \sigma_i^2$. Subsequently, with measurements from $N \geq 3$ non-collinear BTSs, a weighted nonlinear least-squares (WNLS) estimator may be used to estimate $r(k)$ and $c\delta t(k)$. 
IV. CONCLUSION

This paper studied the relative stability of BTS clocks in cellular systems. Data collected over 24 hours showed that BTS clock biases are dominated by a common time-varying bias. This permits re-parametrization of pseudorange measurements by one common clock bias, allowing a receiver to navigate exclusively using cellular signals in the case of GNSS loss. A navigation framework that leverages the common bias re-parametrization was developed and a stochastic auto-regressive model of the deviations from the common bias was proposed.

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